Trajectory Planning Optimization with Dynamic Modeling of Four Wheeled Omni-Directional Mobile Robots

Ehsan Hashemi, Maani Ghaffari Jadidi, Omid Bakhshandeh Babarsad

***Abstract*—Path planning together with the tuning and determination of controller parameters are major concerns in omnidirectional mobile robots. Defining appropriate controller parameters in acceleration and deceleration to reach far and near target points without slippage is one of critical issues since some troubles due to unregulated velocities may greatly affect the ability of robot for the specified path planning and attaining the mentioned targets. A robot accurate kinematic and dynamic modeling and simulation accompanied by velocity and acceleration filtering are mainly discussed in this paper. Major changes and improvements in motion analysis, simulation and accuracy for the newly presented model and its efficiency are discussed in comparison with the previous simple kinematic modeling. Employing the new approach for robot dynamic modeling, particularly acceleration filtering, results in to the more precise robot control and achieving appropriate results.**

I. Introduction

Omni-directional mobile robots have the ability to move concurrently and independently in rotation and movement in the plane. Kinematic and dynamic equations besides reliable controller are the main approach in many researches on these robots, but there is a significant drawback with some traditional controllers for localization when the distance to target has a wide variety and surface condition changes. An accurate trajectory tracking control which falls into two different duties, path planning and trajectory following, is the key part for various applications of omnidirectional robots [1, 2, 3]. Omni-directional mobile robots are widely studied for dynamic environmental applications because of their swiftness and reasonable accuracy for diverse in-plane maneuvers and trajectory following [3-6]. The trajectory control is basically performed by first building a geometric path and then by using feedback control to track the path. For rapidly evolving environments, the dynamic abilities of the mobile robots shall be considered and implemented [1]. In [1, 7, 8], kinematic and dynamic models of omnidirectional mobile robots are presented properly and the robot dynamic model is simplified as a linear system. This modeling contains dynamic behavior of drivers, but the nonlinear coupling between the rotational and translational maneuvers has not been analyzed. In [1, 9, 7], control strategies and optimal path planning have been developed for position control without consideration of orientation control, and the designed controller was tested in simulations and experiment. Two independent PID controllers are designed for controlling position and orientation separately in [8] based on the simplified linear model. A nonlinear dynamic model including the nonlinear coupling terms was introduced in [10-13]. In [12], based on the same model in [11], PID, self tuning PID, and fuzzy control of omnidirectional mobile robots have been studied. A newly developed and experienced method for filtering velocities and robot dynamic model simulation together with the main drawbacks of previous model are illustrated in the presented paper.

Manuscript received August 31, 2009. This work was supported in part by the Mechatronics Research Laboratory, MRL, Qazvin Islamic Azad University.

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TABLE I
NOMENCLATURE

Symbol	Quantity	SI
B	geometrical matrix	--
c_m	coefficient of damping of the driver	[N.s/m]
c_L	coefficient of damping of the wheel	[N.s/m]
E	drivers voltage matrix, $[E_1\ E_2\ E_3\ E_4]^T$	[V]
EM	driver's back EMF	1/285 V/rpm
F_t	traction force matrix $[f_1\ f_2\ f_3\ f_4]$	[N]
F_m	force and moment on the robot in the moving frame $[F_{mx}\ F_{my}\ T_z]$	[N], [N. m]
F_g	force and moment on the robot in global coordinates $[F_x\ F_y\ T_z]$	[N], [N. m]
J_L	wheel's inertia	2.158e-5 Kg.m^2
J_m	driver's rotor inertia	1.35e-5 Kg.m^2
J	robot's inertia	0.013 Kg.m^2
k_m	driver's torque constant	33.5 mN. m/A
l	wheel base	0.0824 m
m	robot's mass	3.678 Kg
n	gears ratio	4.091
R	driver's terminal resistance phase to phase	0.978 Ω
${}^g_m R$	Rotation matrix	--
r	wheel radius	0.0285 m
T_z	Applied moment on the robot about vertical axis	[N.m]
X_m	local position matrix, $[x_m\ y_m\ \phi]^T$	[m]

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X_g	global position matrix, $[x_g\ y_g\ \phi]^T$	[m]
$\dot{X}_g, \dot{X}_m$	global and Local velocity	[m/s]
$\ddot{X}_g, \ddot{X}_m$	global and Local acceleration	[m/s^2]
$\dot{\phi}$	Angular velocity of the robot	[rad/s]
θ	angle of wheels respect to local coordinates	[Degree]
τ_m	drivers torque matrix, $[\tau_{m1}\ \tau_{m2}\ \tau_{m3}\ \tau_{m4}]^T$	[N. m]
ω_m	angular velocity matrix of the drivers, $[\omega_{m1}\ \omega_{m2}\ \omega_{m3}\ \omega_{m4}]^T$	[rad/s]
$\dot{\omega}_m$	angular acceleration matrix of the drivers	[rad/s^2]
ω_L	Angular velocity matrix of the wheels, $[\omega_{L1}\ \omega_{L2}\ \omega_{L3}\ \omega_{L4}]^T$	[rad/s]
$\dot{\omega}_L$	Angular acceleration matrix of the wheels	[rad/s^2]

II. Motion Control and Robot Model

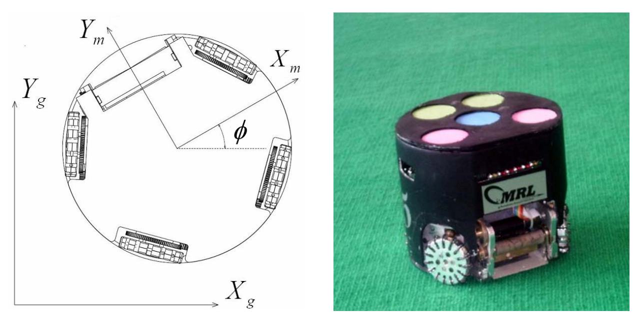
Several works have been performed to develop a procedure for tuning systems and selecting the suitable feedback [14], [15]. The right sequence of local actions for the robot will be recognized using an appropriate path planning which employs all available overall data in non real time. The presented model and dynamic simulation together with the experimental results are focused on a four-wheeled omni-directional robot of MRL RoboCup SSL team which is electrically actuated with four 50 Watts Maxon brushless motors equipped with mechanically driven shaft encoder as shown in figure 1-b. The input signals to the motors are four PWM voltages and the main purpose is to control the robot position and its orientation. In this section, the robot model equations are presented and developed in two parts.

In the first part, kinematic equations of the robot and in the second part dynamic equations based on some typical simplifying assumptions are derived. Many environmental influencing factors have not been considered in the previous model due to complexity. As an instance, the wheels might slip on the floor while floor characteristics had not been completely considered in the motion equations of the earlier model. These drawbacks are removed in the lately developed model with the main aim of drivers torque control. As it is assumed in [3], the wheels have no slippage in the direction of traction force. Only viscous friction forces on the motor shaft and gear are considered. The wheel contact friction forces that are not in the direction of traction force and the motor electrical time constant are neglected in the previous models, but considering the mentioned parameters leads to acceptable results comparing with experimental data. It is shown here that filtering of velocity and acceleration compensate the friction changes and the robot has less slippage. There are two coordinate frames used in the modeling: the body frame which is shown as subscript m and the global frame shown by subscript g. The body frame is fixed on the moving robot with the origin on the robot's geometric center assumed to be the center of gravity, as depicted in figure 1-a. The global frame is also fixed on the field of play. Proposed simulation is developed by MATLAB Simulink and Runge-Kutta 4 is selected as the proper method to solve the robot EOM. The simulated model behaves very similar to the real robot in sensible input range as will be discussed in section ***V***. Implementing the angular velocity obtained by encoder outputs and designing a PID controller via Ziegler-Nichols tuning method lead to a desirable motor velocity control. Although noise, saturation and dead zone make some perturbation in velocity control, robustness of designed PID controller compensates most of them.

A. Kinematic Modeling

A reference-tracking objective and making the output follow the reference or set point are satisfied with standard block diagram. Kinematic equations and relations between robot velocity and various geometry specifications are expressed in the following section. Robot position and orientation shown in figure 1-b in moving and global coordinates are:

$$\begin{cases} X_m = [x_m \quad y_m \quad \phi]^T \\ X_g = [x_g \quad y_g \quad \phi]^T \end{cases} \tag{1}$$



(a) (b)

Fig. 1. (a) Local and Global coordinates; (b) MRL SSL robot for RoboCup 2009

Rotation matrix ${}^g_m R$ used to change the coordinates from moving to global is expressed as:

$${}^g_m R = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

After taking time derivative of the robot position, velocity vector in the global coordinates is obtained as shown in equation (3). Velocity vector which related to the wheel geometry and angular velocity in the global coordinates as:

$$\dot{X}_g = {}^g_m R \dot{X}_m = {}^g_m R \left(B^T\right)^{-1} r\omega_L \tag{3}$$

In which Geometrical matrix B is:

$$B = \begin{bmatrix} -\sin\theta_1 & -\sin\theta_2 & -\sin\theta_3 & -\sin\theta_4 \\ \cos\theta_1 & \cos\theta_2 & \cos\theta_3 & \cos\theta_4 \\ l & l & l & l \end{bmatrix} \tag{4}$$

Robot acceleration vector is mathematically described as equation (5) with taking time derivative of equation (3):

$$\ddot{X}_g = {}^g_m\dot{R}\left(B^T\right)^{-1} r\omega_L + {}^g_mR\left(B^T\right)^{-1} r\dot{\omega}_L \quad (5)$$

Angular velocity matrix components, robot geometry and gear ratio are used in calculation of the robot's angular velocity.

B. Dynamic Modeling

The nonlinearities, like motor dynamic constraints or friction related to the robot's velocity, can greatly affect the robot's behavior, especially when it is accelerated and decelerated [16]. The most important purpose of the following equations is developing relations between driver's torque and actuating voltage, angular velocity and angular acceleration to describe the robot's dynamic behavior. Applied force matrix in the local coordinates and traction force matrix are related as follow:

$$F_m = BF_t \quad (6)$$

Traction force matrix for a 4-wheel omnidirectional robot is:

$$F_t = \begin{bmatrix} f_1 & f_2 & f_3 & f_4 \end{bmatrix}^T \quad (7)$$

In which f_i is the traction force values of wheels. Applied forces and moment matrix could be described as:

$$F_m = \begin{bmatrix} F_{mx} & F_{my} & T_z \end{bmatrix}^T \quad (8)$$

Robot traction force is calculated from equation (10) which represents the dynamic equation:

$$F_t = B^{-1}\, {}^g_mR^T m\left[{}^g_m\dot{R}\left(B^T\right)^{-1} r\omega_L + {}^g_mR\left(B^T\right)^{-1} r\dot{\omega}_L\right]^T \quad (9)$$

The required torque of the driver concerning all loads and motors rotational inertias is:

$$\begin{aligned}\tau_m =& \left[\left(J_m + \frac{J_L}{n^2}\right)I_{4\times4} + \frac{r^2}{n^2}B^{-1}\,{}^g_mR^T m\,{}^g_mR\left(B^T\right)^{-1}\right]\dot{\omega}_m \\ &+ \left[\left(c_m + \frac{c_L}{n^2}\right)I_{4\times4} + \frac{r^2}{n^2}B^{-1}\,{}^g_mR^T m\,{}^g_m\dot{R}\left(B^T\right)^{-1}\right]\omega_m\end{aligned} \quad (10)$$

Equation (10) could be simplified as the following equation in which Z and V matrix elements are listed in Appendix I.

$$\tau_m = Z\dot{\omega}_m + V\omega_m \quad (11)$$

Z and V matrices are shown in equations (13) and (14)

$$Z = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \\ k_2 & k_1 & k_4 & k_3 \\ k_3 & k_4 & k_5 & k_6 \\ k_4 & k_3 & k_6 & k_5 \end{bmatrix} \quad (12)$$

$$V = \begin{bmatrix} k_7 & -k_8\dot{\phi} & k_9\dot{\phi} & k_{10}\dot{\phi} \\ k_8\dot{\phi} & k_7 & -k_{10}\dot{\phi} & -k_9\dot{\phi} \\ -k_9\dot{\phi} & k_{10}\dot{\phi} & k_7 & -k_{11}\dot{\phi} \\ -k_{10}\dot{\phi} & k_9\dot{\phi} & k_{11}\dot{\phi} & k_7 \end{bmatrix} \quad (13)$$

Driver's dynamic behavior considering its actuating voltage, back EMF, terminal resistance, and the motor torque constant is also demanded for next stages to simulate the torque control method. It is written as:

$$\tau_m = \left(E - EM.\omega_m\right)\frac{k_m}{R} \quad (14)$$

Combining equation (11) with equation (14) results into the coupled nonlinear motion equation of four-wheeled omni-directional mobile robots with the following expression:

$$E = \left(\frac{R}{k_m}Z\right)\dot{\omega}_m + \left(\frac{R}{k_m}V + EM.I_{4\times4}\right)\omega_m \quad (15)$$

Equation (15) relates driver specifications and robot's dynamic characteristics, thus plays an essential role for simulation.

III. Velocity and Acceleration Filtering

Effective checking of specified maneuver velocity and acceleration by velocity and acceleration filtering is required to reach a precise path planning algorithm as it is approved by experimental results. This filtering will be done after extracting maximum velocity and acceleration from related curves as described here. Dynamic model with considering all physical parameters and ground conditions are used for presenting a relatively useful slip-free model. This model is executed for so called velocity and acceleration cone method [13], and then the maximum accessible velocity and acceleration are extracted. The main advantage of this method is that ground physical conditions such as friction factor and robot's physical parameters could easily change and modified. Therefore, tunings are simply performed with the new conditions.

A. Velocity Filtering

As described in literatures [13] for three-wheel robots, the robot speed at any point on the path is within its speed limitation corresponding to the rotational speed. It is also concluded that the robot acceleration at any point on the path is within its acceleration constraint corresponding to the robot rotational acceleration. Therefore, $V_{\max}$ and $\dot{\phi}_{\max}$ are calculated implementing the so called velocity cone method, kinematic, and dynamic constraints of omni-directional mobile robots. The novel proposed filtering method in this paper is generated for four-wheel robots with normalizing the velocity components. V_{input} which is calculated by the game strategy controller and artificial intelligence sections for reaching specified targets and moving through various paths, is compared by the estimated $V_{\max}$. If the estimated value for maximum speed is greater than the input desired velocity, all horizontal and vertical components of desired velocity could be executed for the next steps and filtered value of robot's rotational velocity, $\dot{\phi}_f$ is calculated with the components of desired velocity. For this condition if the desired robot rotational velocity, $\dot{\phi}_{input}$ is smaller than $\dot{\phi}_f$, $\dot{\phi}_{input}$ is employed for the next calculations. $\dot{\phi}_f$ will be used if the desired rotational velocity has the larger value than the filtered one. In contrast, if the estimated value for maximum speed is smaller than the input desired velocity, filtering should be performed using normalizing the desired velocity and multiplying by the maximum one which results into a new linear velocity. Equation (16) represents relation between the maximum linear and angular velocities as extracted from the velocity cone method for the particular robot's model.

$$\dot{\phi}_m + \left(\frac{44.217}{3.159}\right)\|V_m\| = 44.217 \tag{16}$$

Practical simulated maximum linear and angular velocity obtained by method mentioned above are

$V_{\max} = 3.159 \ m/s$ and $\dot{\phi}_{\max} = 44.217 \ rad/s$

B. Acceleration Filtering

Restricting the maximum acceleration assigned to the motion can reduce the slippage as expressed in [13]. The same method for determining the maximum permissible linear and rotational velocity and its filtering is employed to generate a relatively slip free motion with the filtered acceleration in different maneuvers, but the main parameters are $a_{\max}$ and a_{input} in the calculation described in the previous section. Maximum acceleration approximated modified by dynamic limitations and robot parameters is $a_{\max} = 0.826 \ m/s^2$ The maximum acceleration is achieved by friction coefficient about 0.19 and it could be different in diverse fields and robot's maneuver.

IV. Path Planning and AI

Various mobile robots are equipped with Rapidly Random Tree planner, RRT, for suitable obstacle avoidance and swift motion to attain specified targets. *Extend*, *Distance*, and *Random State* are the main functions of the RRT planner.

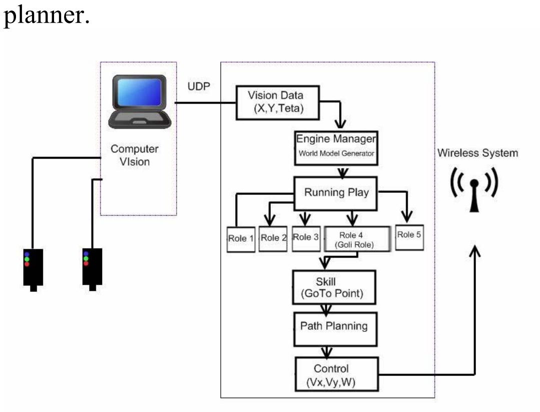


Fig. 2. MRL AI sections and relations for velocity control

New states that could be reached from the target are calculated by *Extend* function utilizing specified distance parameters. If a collision with an obstacle occurs during moving to that new state in the environment, a default value, *Empty* state, will be returned which represents the collision. Next Distance between the current and the goal state is measured by *Distance* function to find out if the current state is near enough to the *Goal*. *Random State* also returns a new random location.

Localization is utilized by a machine vision package and communication with the AI section. AI machine in cooperation with the developed software to control robots are shown in figure 2 which performs by bidirectional wireless connection in 73 packets per second for each robot.

V. Results and Discussion

The proposed filtering method was utilized in four-wheeled omni-directional SSL robot and simulation results for horizontal and vertical movements are graphically illustrated in figures 3-a and 3-b. This compares new approach open and closed loop simulation with the desired path decided by the path planning section of the model. These paths show responses of dynamically simulated model in the horizontal and vertical maneuvers which predicts an error with maximum value about 29% and 21% for the first sharp edge in horizontal and vertical movement respectively with the open loop, OL simulator. These values changed to 4.3% and 6.2% for the closed loop regulated PID controller, CL. These curves are predicted by the maximum attainable velocity with a slip-free motion in the simulator which is 3.159 m/s.

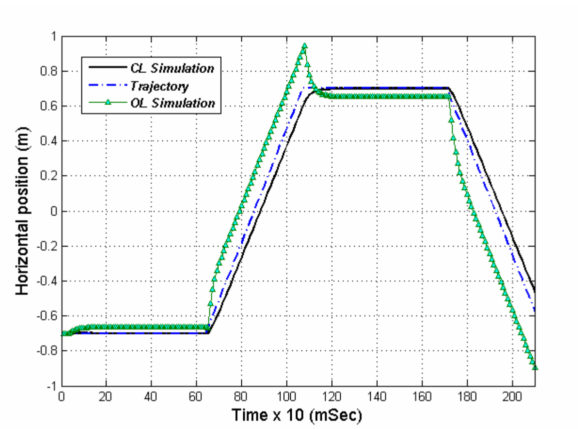


Fig. 3-a. Dynamic simulation results in horizontal maneuvers

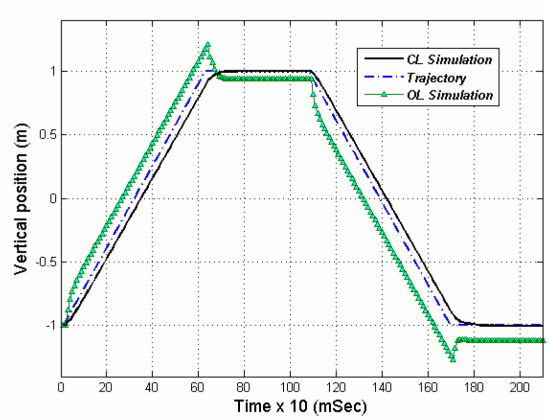


Fig. 3-b. Dynamic simulation results in vertical maneuvers

Robot's angular velocity is controlled and modified by the mentioned model which leads to different values in assessment with the traditional PID controllers. Results are evaluated with the desired increase of angular velocity with the ramp shape for both the OL and CL controllers in figure 4. This increase in angular velocity is considered in the rectangular path maneuvers as requested by the path planner. In figure 5 similar circular paths are assigned to the conventional controller without the dynamic simulation and the novel one.

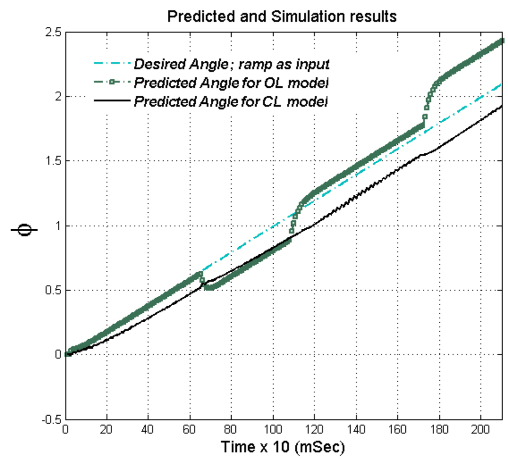


Fig. 4. Simulator prediction for robot's ramp-shaped rotation

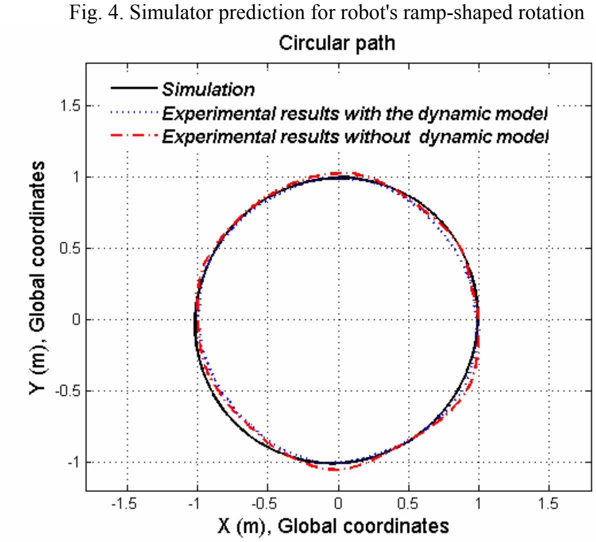


Fig. 5. Simulator prediction and experimental results for both system with and without the newly proposed model

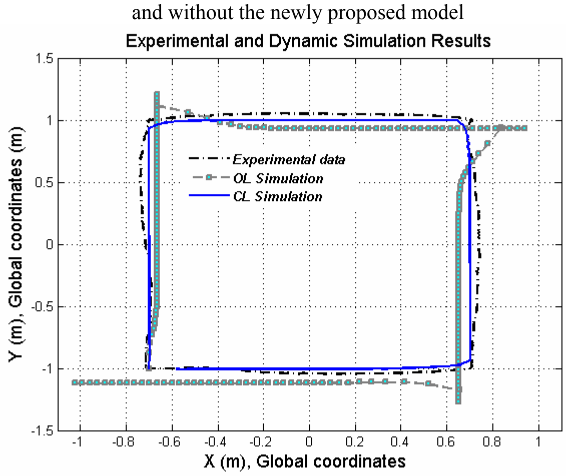


Fig. 6. Simulator prediction and experimental results for rectangular path

This difference is one of the most significant characteristics required in the soccer robot's motion.

Rectangular trajectory desired for the 4-wheeled omni-directional mobile robot by the maximum slip-free velocity is compared with the simulator predicted path which results into the maximum errors 8.1% for vertical and 3.7% for horizontal position in the CL model. Experimental and the presented simulation data accomplished by the new model are graphically compared in figure 6. Comprehensive evaluation of the system on which the modified PID controller and the conventional controller are installed is shown in figure 7.

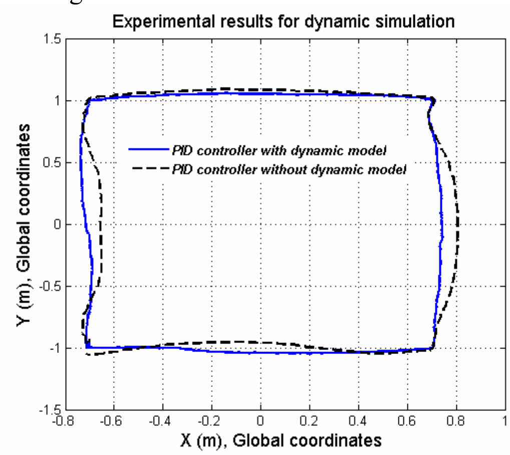


Fig. 7. Experimental data for both the conventional and new controllers

Absolute mean errors for three zones are tabulated and compared for OL and CL simulation with the rectangular trajectory in the global coordinates as Table II. The error values are in percent for speeds lower and higher than and equal to the maximum attainable speed without slip. Zone I is defined for x=0.7m and vertical movement of robot between -1m and 0.58m. Zone II consists of the first perpendicular corner of trajectory with x position between -0.7m and -0.34m and y position between 0.58m and 1m. Zone III is classified as the horizontal path after the corner with y=1m and 0.34m<x<0.7m. All CL results are more practical and better than the OL predicted errors except one value which is for V=4m/s and around the first right angle corner which is due to the higher velocity and could be modified by regulating PID coefficients or using adaptive PID controllers.

Table II
ABSOLUTE MEAN ERRORS FOR OL AND CL CONTROLLERS

	Open Loop Simulation			**Closed Loop PID Simulation**		
	Zone I	Zone II	Zone III	Zone I	Zone II	Zone III
V=2 m/s	5.36%	7.03%	5.19%	2.47%	2.59%	2.27%
V=3.15 m/s	8.65%	10.04%	8.54%	4.17%	4.88%	3.11%
V=4 m/s	10.12%	13.65%	10.59%	9.06%	21.43%	9.41%

VI. CONCLUSION

Introducing velocity and acceleration filters which enables the controller to exactly control the input current and voltage of drivers to prevent slip, will improve the efficiency while experimental results approve this issue too. Robot's angular velocity increase during the rectangular trajectory shows a smooth shift around perpendicular edges in CL model and results in to a maximum 13.2% error from the preferred value while OL simulation predicts maximum 19.5% error. As it is concluded from the simulated curves for the rectangular trajectory and the highest achievable slip-free speed, the maximum horizontal position error in vertical paths for the modified PID controller is about 6.7% and 5.3% for OL and CL model respectively, but this error for the conventional controller is about 13.8% which shows a reasonable method with good accuracy. All above mentioned errors are maximum errors and the absolute mean ones are lower as they are listed in table II. In addition, fluctuations for the simple controller are more around the sharp edges and changing direction maneuvers. The greatest vertical position errors in horizontal movement are 8.2 % and 5.1 % for the traditional and the proposed model correspondingly. Mass transformation in acceleration and deceleration maneuvers is not considered in this paper, but practical results show acceptable approximation. This transformation and its effect on path planning are authors' next approaches and research subjects for omni-directional robots motion analysis. Adaptation of the maximum acceleration to filter acceleration in the fields with variable coefficients of friction especially in diverse directions and areas is the authors' next research topics.

APPENDIX

Z and V matrix elements shown in equation (12) and (13) are mathematically described as bellow. These parameters introduce relationship among various dynamic characteristics of the robot to analyze the robot's dynamic behavior.

$$k_1 = J_m + \frac{J_L}{n^2} + \frac{r^2}{n^2}\left(0.2810m + 11.75J\right)$$
$$k_2 = \frac{r^2}{n^2}\left(0.03812m + 11.75J\right)$$
$$k_3 = \frac{r^2}{n^2}\left(-0.2619m + 9.050J\right)$$
$$k_4 = \frac{r^2}{n^2}\left(-0.05717m + 9.050J\right)$$
$$k_5 = J_m + \frac{J_L}{n^2} + \frac{r^2}{n^2}\left(0.2459m + 6.9710J\right) \quad \text{(A-1)}$$
$$k_6 = \frac{r^2}{n^2}\left(0.07323m + 6.9710J\right)$$
$$k_7 = c_m + \frac{c_L}{n^2} \quad k_8 = 0.2784\frac{r^2}{n^2}m \quad k_9 = 0.02184\frac{r^2}{n^2}m$$
$$k_{10} = 0.2566\frac{r^2}{n^2}m \quad k_{11} = 0.2347\frac{r^2}{n^2}m$$

ACKNOWLEDGMENT

Authors gratefully acknowledge the technical support of MRL SSL RoboCup team members.

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