PID Tuning Rules for Second Order Systems

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ABSTRACT

This paper presents PID tuning rules for second systems. These tuning rules are derived by optimizing the integrated absolute errors of set point and load disturbance responses under robustness and bandwidth constrains. For deriving the tuning formulas, PID controllers for normalized systems were designed. The relationship between the controller parameters, the parameters that characterize the system dynamics and the normalized gain crossover frequency are determined and the tuning formulas are then derived. Simulation examples and experimental results are provided to demonstrate the effectiveness of these tuning rules.

Key Words: bandwidth, PID controllers, tuning rules, second order system.

1. INTRODUCTION

PID controller is the most common control algorithm and is widely used. There are a lot of tuning rules for PID controllers [1-19]. Most of the tuning rules are derived for process control and are derived under idealize assumptions, such as infinite bandwidth. In fact, in most applications, the measurement noise, the range of manipulated variable and the sample rate of the system limit the closed-loop bandwidth.

In this paper, PID tuning rules for second order systems are derived. For deriving the tuning rules, PID controllers for some normalized second order systems are designed. These PID controllers are designed by optimizing the integrated absolute errors (IAE) of set point and load disturbance responses under constraints on robustness and crossover frequency (the frequency where the loop gain equals one). Note that the closed-loop bandwidth can be approximated by the crossover frequency [20]. Therefore, the PID controllers are designed under bandwidth constrain. When the PID controllers for normalized systems are designed, the curve fitting technique is used to derive simple formulas that describe the relationships among normalized PID controller parameters, parameters that characterize the system dynamics and the normalized crossover frequency. Once these formulas are derived, they can be used to tune the PID controllers. Simulation examples and experimental results show that these formulas give satisfactory results.

This paper is organized as follows: In Section 2, the model of systems and the design method of PID controller are described. Section 3 presents the way to derive the tuning rules. Simulation and experimental results are given in Section 4. Finally, conclusions are drawn in Section 5.

2. MODEL AND DESIGN METHOD

Consider the following system

$$G_s = \frac{K_s\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (1)$$

where K_s is the static gain, ω_n is the undamped natural frequency and ζ is the damping ratio. For over-damped systems ($\zeta > 1$), the models in (1) can be rewritten as

$$G_s = \frac{K_s}{(1+sT)(1+sT_1)} \quad (2)$$

where

$$1/T = \omega_n(\zeta - \sqrt{\zeta^2 - 1}),\ 1/T_1 = \omega_n(\zeta + \sqrt{\zeta^2 - 1}). \quad (3)$$

When $T_1 < 10T$ ($\zeta > 1.74$), the models in (2) can be well approximated by

$$G_{so} = \frac{K_s}{(1+sT)}. \quad (4)$$

In this paper, the systems with $\zeta \le 2$ are considered.

Suppose that the following PID controller is employed to control the systems:

$$u = K\left((by_r - y) + \frac{1}{T_i}\int e\,dt + T_d\frac{de}{dt}\right),$$

$$e = y_r - y. \quad (5)$$

where u, y_r, y, K, b, T_i, T_d and e are the controller output, set point, system output, controller gain, set-point weighting, integral time, derivative time, and error, respectively. Notice that the controller parameters K, b, T_i and T_d must be positive constants.

Denote the loop transfer function of the closed-loop system as $G_l(s)$ and define M_s as

$$M_s = \max_{\omega}\left|\frac{1}{1+G_l(j\omega)}\right|.$$

M_s is the inverse of the shortest distance from the Nyquist curve of the loop transfer function to the critical point –1 and is a measure of stability robustness. Typical value of M_s is in the range from 1.4 to 2.0 and the standard value is 2.0 [10].

Let e_s denote the error caused by a unit step set-point change and e_d denote the error caused by a unit step disturbance at the system input, respectively. Define the performance index as

$$J = \int_0^\infty |e_s(t)| dt + \int_0^\infty |e_d(t)| dt .$$

In this study, the PID controller parameters were chosen such that the performance index J is minimized under the following constraints:

$$K > 0 \, , \, 1 \ge b > 0 \, , \, T_i > 0 \, , \, T_d > 0 \, ,$$

$$M_s \le m \text{ and } \omega_g \le B$$

where constant $m > 1$ represents the minimal requirement of stability robustness (In this study, we let $m = 2.0$.), ω_g denotes the crossover frequency and $B > 0$ represents the upper bound of crossover frequency. Clearly, this is a constrained nonlinear optimization problem. In this study, the genetic algorithm described in [17] is used to solve this optimization problem.

3. THE TUNING RULES

The PID controller design method described in last section is a time consuming procedure. If we can find simple formulas that describe the relations among the parameters of PID controller, the parameters that characterize the system dynamics and the upper bound of crossover frequency, the user can obtain proper PID controller parameters easily and need not to run the entire design procedure. In this section, the way to derive the tuning rules will be described.

The closed-loop transfer function of the system G_s controlled by the PID controller described in (5) is

$$G_{y_r\omega} = \frac{K(b + \frac{1}{T_i s} + T_d s)(\frac{K_s \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2})}{1 + K(1 + \frac{1}{T_i s} + T_d s)(\frac{K_s \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2})}$$

$$= \frac{\overline{K}(b + \frac{1}{\overline{T}_i \overline{s}} + \overline{T}_d \overline{s})\overline{G}_s(\overline{s})}{1 + \overline{K}(1 + \frac{1}{\overline{T}_i \overline{s}} + \overline{T}_d \overline{s})\overline{G}_s(\overline{s})} \qquad (6)$$

where $\overline{K} = KK_s$, $\overline{T}_i = T_i\omega_n$, $\overline{T}_d = T_d\omega_n$ are the normalized controller parameters, $\overline{s} = s/\omega_n$ and

$$\overline{G}_s = \frac{1}{(\overline{s}^2 + 2\zeta\overline{s} + 1)}$$

is the normalized system model. Systems with the same $\overline{K}$, $\overline{T}_i$, $\overline{T}_d$, b and ζ will have similar responses, both in time and frequency domain. The only difference is in the scale of time and frequency axis (scaled by ω_n). For convenience, the upper bound B is normalized as $\overline{B} = B/\omega_n$. If we can find the relations between ζ, $\overline{B}$ and the normalized control parameters, and represent $\overline{K}$, $\overline{T}_i$, $\overline{T}_d$ and b as functions of ζ and $\overline{B}$, these functions can be used to tune the PID controllers systems that modeled by G_s.

For deriving the tuning rules, the PID controllers for the normalized systems $\overline{G}_s$ with ζ=0.1, 0.2, … 2.0 and normalized upper bound $\overline{B}$ = 1, 2, … 10 were designed. Then the normalized controller parameters were plotted as functions of ζ and $\overline{B}$. We then utilized curve fitting technique to find the relations between the normalized controller parameters, ζ and $\overline{B}$.

Fig. 1 shows the designed results of normalized systems. It was tried to express the normalized controller gain as

$$\overline{K} = f(\zeta, \overline{B})$$

and analogous expressions for other parameters. By the data in Fig. 1, it can be found that the normalized gain $\overline{K}$ increases rapidly as $\overline{B}$ increases and the variation of $\overline{T}_i$, $\overline{T}_d$ and b are large for $\overline{B} \le 2$. This makes it difficult to do curve fitting. In order to obtain better fitting, the data is separated into two groups ($\overline{B} \le 2$ and $10 \ge \overline{B} > 2$) for curve fitting. After some trials, we found that for $10 \ge \overline{B} > 2$ the function of $\overline{K}$, $\overline{T}_i$, $\overline{T}_d$ and b could be well approximated by function of the form

$$f(\zeta, \overline{B}) = a_0 + a_1\zeta + a_2\zeta^2 + \overline{B}(a_3 + a_4\zeta + a_5\zeta^2) + \overline{B}^2(a_6 + a_7\zeta + a_8\zeta^2) + \overline{B}^3(a_9 + a_{10}\zeta + a_{11}\zeta^2) \qquad (7)$$

Table 1 shows the coefficients a_0, a_1, … a_{11} of the functions of the form as in (7) that were least squares fitted to the data in Fig. 1.

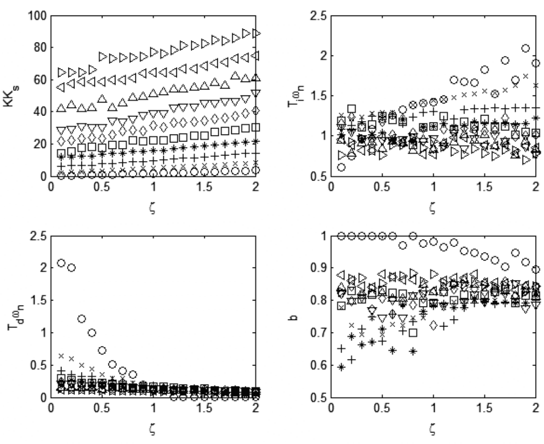

Fig. 1. Design results for normalized control system $\overline{G}_s$. ($\overline{B} = 1$: o, $\overline{B} = 2$: x, $\overline{B} = 3$: +, $\overline{B} = 4$: *, $\overline{B} = 5$: , $\overline{B} = 6$: ◊, $\overline{B} = 7$: ∇, $\overline{B} = 8$: Δ, $\overline{B} = 9$: <, $\overline{B} = 10$: >)

For deriving the tuning rules for $\overline{B} \le 2$, more controllers were designed. Fig. 2 depicts the designed results. After some trial, we found that it was difficult to fit the data in Fig. 2 reasonably well to a function like (7), especially for $\overline{T}_d$. Therefore, the curves in Fig. 2 were treated separately for each value of $\overline{B}$ and express the normalized controller parameters as functions of ζ. Table 2 lists the results of fitting.

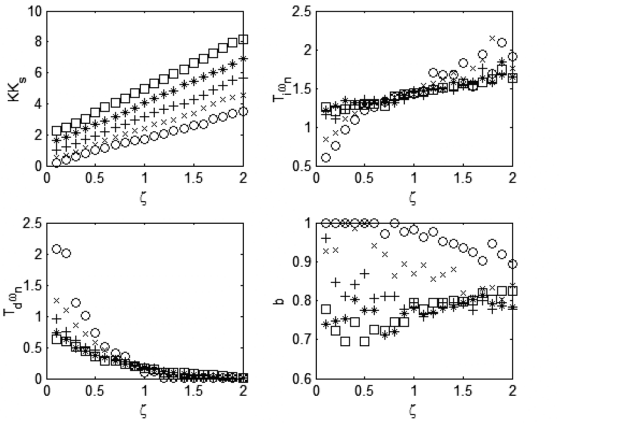

Fig. 2. PID controller design results for normalized systems $\overline{G}_s$ for $\overline{B}=1$(o), $\overline{B}=1.25$(x), $\overline{B}=1.5$(+), $\overline{B}=1.75$(*) and $\overline{B}=2$( ).

4. EXAMPLES

In order to demonstrate the performance and use of the tuning rules, these tuning rules were applied to a few systems. Comparisons will be made with Astrom and Hagglund's method [10, 18], Haeri's method [19] and Shen's [17] method.

In the examples, the model of system was determined by step and relay feedback test [10]. Step test can determine the static gain K_s of G_s, while a relay feedback test (with an integrator inserted between the relay and the system under test) can determine the ultimate gain K_u and ultimate frequency ω_u of G_p/s. By the definition of ultimate gain and frequency, ω_n and ζ can be obtained as

$$\omega_n=\omega_u,\ \zeta=\frac{K_sK_u}{2\omega_u}. \tag{8}$$

Example 1: Consider a under damped system

$$G_1(s)=\frac{3}{s^2+s+3}$$

A step test obtained $K_s=1$. ω_n and ζ were determined by relay feedback test as 1.73 and 0.288 respectively. Applying the exact parameters to Haeri's method [19] gives $K=8.383$, $T_i=8.45$, $T_d=0.929$ and $b=1$. The crossover frequency of $G_1(s)$ controlled with this PID controller is 23.48 and the closed-loop bandwidth is about 23 (about thirteen times wider than that of $G_1(s)$). Let $\overline{B}=3.5$ and apply the parameters of approximated model to the proposed tuning rules, the following controller parameters can be obtained: $K=8.55$, $T_i=0.67$, $T_d=0.226$, and $b=0.76$. The gain of this controller is similar to that tuned by Haeri's method. The crossover frequency of $G_1(s)$ controlled with this PID controller is 6.23 and the closed-loop bandwidth is about 8.9.

Fig. 3 depicts the control result of these two controllers. Clearly, the performance of the proposed method is better than Haeri's method.

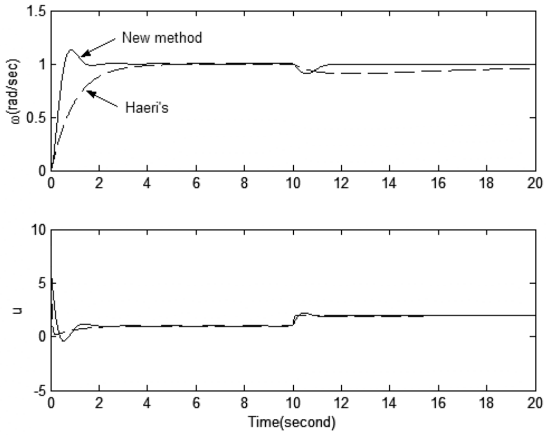

Fig. 3. Set point and load disturbance responses of $G_2(s)$ controlled by a PID controller tuned by the proposed method and Haeri's method.

Example 2: Consider a system

$$G_2(s)=\frac{1}{(1+s)(1+0.2s)}$$

A step test obtained $K_s=1$. ω_n and ζ were determined by relay feedback test as 2.16 and 1.318 respectively. For comparison, the tuning rules proposed by Astrom and Hagglund [10], and Shen [17] also used to tune the PID controller. For their method, the approximated model $e^{-0.105s}/(1+1.11s)$ was used. Using this model, Astrom and Hagglund's rule gives $K=40.52$, $T_i=0.29$, $T_d=0.076$, and $b=0.23$. The crossover frequency of the closed-loop system with this controller is 16.88 and the bandwidth is 22. Shen's method gives $K=79.55$, $T_i=0.51$, $T_d=0.08$, and $b=0.75$. The crossover frequency of the closed-loop system with this controller is 33 and the bandwidth is 38. Select $\overline{B}=7$, the proposed method gives $K=42.73$, $T_i=0.45$, $T_d=0.061$, and $b=0.84$. The bandwidth of the closed-loop system with this controller is similar to that tuned by Astrom and Hagglung's method.

Fig. 4 shows the set point and load disturbance responses of $G_2(s)$ controlled by the controllers derived above. Shen's method provides best performance. But the bandwidth of the closed-loop system is much beyond the bandwidth of $G_2(s)$. With similar bandwidth, the proposed method provides better performance than Astrom and Hagglung's method.

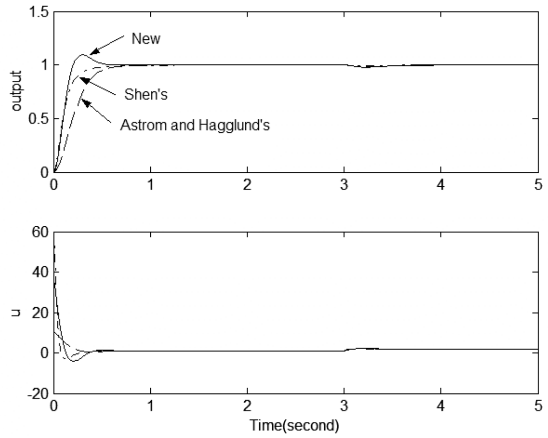

Fig. 4. Set point and load disturbance responses of $G_2(s)$ controlled by a PID controller tuned by the proposed method, Astrom and Hagglund's method, and Shen's method.

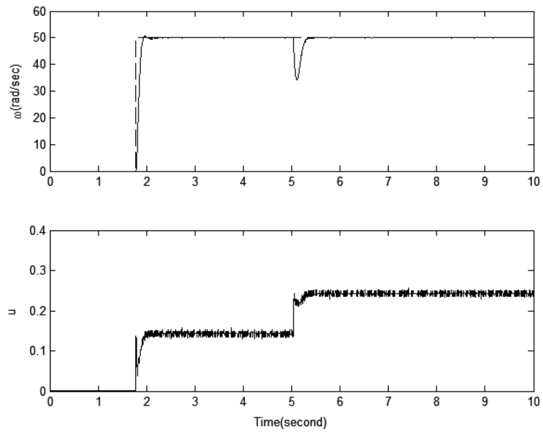

Fig. 5. Set point and load disturbance responses of the system in example 3 controlled by the PID controller tuned by the proposed method.

Experimental results: In order to show the applicability of the proposed tuning rules, experiments were carried out. In the experiments, the PID controller was implemented with back-calculation based anti-windup [10] and a low-pass filter with time constant $T_d/10$ was connected to the derivative part. The input of this low-pass filter was connected to system output directly. That is, the derivation acted on filtered system output directly. Moreover, the sampling frequency was chosen to be 200Hz. In the following, the results on a motor speed control systems are presented.

Example 3: Consider a disk that was driven by DC motor with a flexible shaft. In the experiment, the speed was estimated by an encoder. This resulted in a measure noise about ± 0.4 rad/sec.

The K_s of this system was obtained by a step test as 350. While ω_n and ζ were estimated as 19.64 and 0.454 respectively.

For speed control, a PID controller was tuned. We hope the fluctuation of controller output keeps inside ± 0.02 (± 0.2 volt) when the system is in steady state. For the implemented PID controller, the high frequency gain from system output to controller output is $(1+10)K$. Therefore, K should satisfy the following inequality:

$$0.4(1+10)K \le 0.02 \Rightarrow K \le 0.0045$$

Selecting $\overline{B}=1$, the controller parameters can be obtained as $K = 0.0024$, $T_i = 0.055$, $T_d = 0.052$, and $b = 0.95$. Fig. 5 shows the controlled result of this controller. As the results shown, the performance is good.

5. CONCLUSIONS

In this paper, PID tuning rules for second order systems are proposed. These tuning rules take the bandwidth limitation into consideration. Therefore, the user can tune the PID controller according to the bandwidth limitation of the system. Simulation examples and experiment are provided to demonstrate the performance and the use of the proposed tuning rules.

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REFERENCES

[1] Ziegler, J. G. and N. B., Nichols, "Optimal settings for automatic controllers", ***Trans. ASME,*** Vol. 64, pp. 759-768 (1942).

[2] Cohen, G. H. and G. A., Coon, "Theoretical consideration of retarded control", ***Trans. ASME,*** Vol. 75, pp. 827-834 (1953).

[3] Smith, C. A. and C. B., Corripio, ***Principals and Practices of Automatic Control***, Wiley, New York, (1985).

[4] Shinkey, F. G., ***Process Control System Application, Design and Tuning***, 3rd ed., McGraw-Hill, New York, (1988).

[5] Morari, M. and E., Zafiriou, ***Robust Process Control***, Prentice-Hall, Englewood Cliffs, NJ, (1989).

[6] Chien, I. L. and P. S., Fruehauf, "Consider IMC tuning to improve controller performance", ***Chem. Eng. Prog.*** Vol. 86, pp. 33-41 (1990).

[7] Hang, C. C., K. J., Astrom, and W. K., Ho, "Refinements of the Ziegler-Nichols tuning formula", ***IEE Proc.-D: Control Theory Appl.,*** Vol. 138, pp. 111-118 (1991).

[8] Astrom, K. J., T., Hagglund, C. C. Hang, and W. K., Ho, "Automatic tuning and adaptation for PID controllers-A survey", ***IFAC J. of Control Eng. Practice,*** Vol. 1, pp. 699-714 (1993).

[9] Zhaung, M. and D. P., Atherton, "Automatic tuning of optimal PID controllers", ***IEE Proc.-D: Control Theory Appl.,*** Vol. 140, pp. 216-224 (1993).

[10] Astrom, K. J. and T., Hagglund, ***PID Controllers: Theory, Design and Tuning,*** ISA, Research Triangle

Par, NC, (1995).
[11] Ho, W. K., C. C., Hang, and L. S., Cao, "Tuning of PID controllers based on gain and phase margin specification", ***Automatica,*** Vol. 31, pp. 497-502 (1995).
[12] Abbas, A., "A new set of controller tuning relations", ***ISA Trans.*** Vol. 36, pp. 183-187 (1997).
[13] Ho., W. K., C. C., Hang, and J., Zhou, "Self-tuning PID control of a plant with under-damped response with specifications on gain and phase margins", ***IEEE Trans. Control Syst. Technol.,*** Vol. 5, pp. 446-452 (1997).
[14] Wang, Q. G., T. H., Lee, H. W., Fung, Q., Bi, and Y., Zhang, "PID tuning for improved performance", ***IEEE Trans. Control Syst. Technol.,*** Vol. 7, pp. 457-465 (1999).
[15] Shen, J. C., "New tuning method for PID control of a plant with under-damped response", ***Asian Journal of Control***, Vol. 2, pp. 31-41 (2000).
[16] Shen, J. C., "Fuzzy neural networks for tuning PID controller for plants with under-damped responses", ***IEEE Trans. On Fuzzy Systems,*** Vol. 9, pp. 333-342 (2001).
[17] Shen, J. C., "New tuning method for PID controller", ***ISA Trans***., Vol. 41, pp. 473-484 (2002).
[18] Hagglund, T., and K. J., Astrom, "Revisiting the Ziegler-Nichols tuning rules for PI control", ***Asian Journal of Control,*** Vol. 4, 364-380 (2002).
[19] Haeri, M., "Tuning rules for PID controller using a DMC strategy", ***Asian Journal of Control,*** Vol. 4, pp. 410-417 (2002).
[20] Franklin, G. F., J. D., Powell, and A., Emami-Naeini, ***Feedback Control of Dynamic Systems***, Addison-Wesley, (1986).

Table 1. Tuning formula for systems that can be modeled by G_s with $2 < \overline{B} \le 10$.

$0 < \zeta \le 2,\ 2 < \overline{B} \le 10$				
	KK_s	$T_i\omega_n$	$T_d\omega_n$	b
a_0	1.8476	1.0743	1.4274	0.8712
a_1	-6.7604	0.7686	-1.8460	-0.1955
a_2	2.8846	-0.0150	0.5692	0.1043
a_3	-0.8778	0.0512	-0.5047	-0.1514
a_4	5.7533	-0.3071	0.7723	0.2142
a_5	-1.9453	-0.0036	-0.251	-0.0828
a_6	0.6445	-0.01	0.0703	0.0339
a_7	-0.7925	0.0329	-0.1107	-0.0454
a_8	0.4080	0.0045	0.0365	0.0168
a_9	0.0071	0.0002	-0.0033	-0.0019
a_{10}	0.0414	-0.0008	0.0052	0.0027
a_{11}	-0.0248	-0.0005	-0.0017	-0.0010

Table 2. Tuning formula systems that can be modeled by G_s with $1 \le \overline{B} \le 2$.

$\overline{B}$	KK_s	$T_i\omega_n$	$T_d\omega_n$	b
1	$1.7034\zeta + 0.0713$	$-0.2382\zeta^2 + 1.1225\zeta + 0.6064$	$2.1266\zeta^2 - 4.6156\zeta + 2.5748$ for $0 < \zeta < 1.2$; $0.0104\zeta^2 - 0.0372\zeta + 0.0376$ for $1.2 \le \zeta \le 2$	$-0.0553\zeta + 1.023$
1.25	$2.149\zeta + 0.2730$	$-0.129\zeta^2 + 0.7975\zeta + 0.8269$	$0.8093\zeta^2 - 2.1177\zeta + 1.4476$ for $0 < \zeta < 1.3$; $0.0232\zeta^2 - 0.0868\zeta + 0.0877$ for $1.3 \le \zeta \le 2$	$-0.089\zeta + 0.9811$
1.5	$2.4511\zeta + 0.7056$	$0.0349\zeta^2 + 0.2337\zeta + 1.1508$	$0.4542\zeta^2 - 1.3187\zeta + 1.0058$ for $0 < \zeta < 1.4$; $-0.0004\zeta^2 + 0.0005\zeta + 0.0052$ for $1.4 \le \zeta \le 2$	$-0.0509\zeta + 0.8618$
1.75	$2.7891\zeta + 1.264$	$0.0135\zeta^2 + 0.2029\zeta + 1.2133$	$0.3205\zeta^2 - 0.9502\zeta + 0.7874$ for $0 < \zeta < 1.5$; $0.3177\zeta^2 - 1.2488\zeta + 1.2260$ for $1.5 \le \zeta \le 2$	$0.0256\zeta + 0.7451$
2	$3.1821\zeta + 1.8282$	$0.0178\zeta^2 + 0.2139\zeta + 1.1847$	$0.2586\zeta^2 - 0.8177\zeta + 0.7209$ for $0 < \zeta < 1.6$; $-0.39\zeta^2 + 1.2784\zeta - 0.9926$ for $1.6 \le \zeta \le 2$	$0.0592\zeta + 0.7083$